Methodological Issues in the Study of Complex Systems

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Abstract

From an engeneering perspective, it is well known that numerous problems hamper the proper control and prediction of complex systems which are essential for the reproducibility of their behavior. In addition, it is not obvious how the concept of complexity ought to be understood within the tradition of physics and its epistemology. Both issues have important ramifications for complex systems in other discplines as well.

Three outstanding topics in this regard are discussed. (1) Many definitions of complexity stress its difference from randomness and are fundamentally context-dependent rather than universal. (2) Complexity measures are often defined in information-theoretical terms, but extend the scope of pure syntax toward semantic and pragmatic dimensions. (3) Mathematical limit theorems, expressing the stability of a result, are often not straightforwardly applicable to complex systems.

1 Introduction

The concept of complexity and the study of complex systems represent an important focus of research in contemporary science. Although one might say that its formal core lies in mathematics and physics, complexity in a broad sense is certainly one of the most interdisciplinary issues scientists of many backgrounds have to face today. Beyond the traditional disciplines of the natural sciences, the concept of complexity has even crossed the border to areas like psychology, sociology, economics and others. It is impossible to address all approaches and applications that are presently known comprehensively here; overviews from different eras and areas of complexity studies are due to Cowan *et al.* (1994), Cohen and Stewart (1994), Auyang (1998), Scott (2005), Shalizi (2006), Gershenson *et al.* (2007), Nicolis and Nicolis (2007), Mitchell (2009), Hooker (2011).

The study of complex systems continues a whole series of interdisciplinary approaches, leading from system theory (Bertalanffy 1968) and cybernetics (Wiener 1948) to synergetics (Haken 1977) and self-organization (Foerster 1962), dissipative (Nicolis and Prigogine 1977) and autopoetic systems (Maturana and Varela 1980), automata theory (Hopcroft and Ullmann 1979), and others. In all these approaches, the concept of information plays a significant role in one or another way, first due to Shannon and Weaver (1949) and later also in other contexts (Zurek 1989, Atmanspacher and Scheingraber 1991, Kornwachs and Jacoby 1996, Marijuàn and Conrad 1996, Boffetta *et al.* 2002, Crutchfield and Machta 2011).

A most important predecessor of complexity theory is the theory of nonlinear dynamical systems, which originated from early work of Poincaré and was further developed by Lyapunov, Hopf, Krylov, Kolmogorov, Smale, Ruelle – to mention just a few outstanding names. Prominent areas in the study of complex systems as far as it has evolved from nonlinear dynamics are fractals (Mandelbrot 1983), chaos (Stewart 1990), cellular automata (Wolfram 1986, 2002), coupled map lattices (Kaneko 1993, Kaneko and Tsuda 2000), symbolic dynamics (Lind and Marcus 1995), self-organized criticality (Bak 1996), computational mechanics (Shalizi and Crutchfield 2001), and network theory (Albert and Barabási 2002, Boccaletti *et al.* 2006, Newman *et al.* 2006).

This ample (and incomplete) list notwithstanding, it is fair to say that one important *open* question is the question for a *fundamental* theory with a *universal* range of applicability, e.g., in the sense of an axiomatic basis, of nonlinear dynamical systems. Although much progress has been achieved in understanding a large corpus of phenomenological features of dynamical systems, we do not have any compact set of basic equations (like Newton's, Maxwell's, or Schrödinger's equations), or postulates (like those of relativity theory) for a comprehensive, full-fledged, formal theory of nonlinear dynamical systems – and this applies to the concept of complexity as well.

Which criteria does a system have to satisfy in order to be complex? This question is not yet answered comprehensively, too, but quite a few essential issues can be indicated. From a physical point of view, complex behavior typically (but not always) arises in situations *far from thermal equilibrium*. This is to say that one usually does not speak of a complex system if its behavior can be described by the laws of linear thermodynamics. (In fact the entire framework of equilibrium thermodynamics may become inapplicable in such situations.) The thermodynamical branch of a system has to become unstable before complex behavior can emerge. In this manner the concept of *instability* becomes an indispensable element of any proper understanding of complex systems.[1]

In addition, complex systems are usually regarded as *open systems*, exchanging energy and/or matter (and/or information) with their environment. Other features which are most often found in complex systems are internal *self-reference* and external boundary conditions such as *control parameters*. Sometimes it is

[1] The contribution by Steinle in this volume addresses the significance of stability for the reproducibility of experimental results in detail.

argued that external boundary conditions gradually become internalized, i.e., become part of the internal dynamics of a system, if one is dealing with living organisms (see, e.g., Atlan 1990). In this context, Maturana and Varela (1980) established the concept of autopoiesis accounting for the fact that living system are able to develop (and modify) their own boundaries. Higher levels in the hierarchy, for instance cognitive or even social systems, raise yet more sophisticated problems, above all the so-called "hard problem" (Chalmers 1995) of how consciousness can be understood in its relation to the physical world.

2 Definitions of Complexity

Problems with standard scientific methodology arise already for definitions of complexity. Subsequent to algorithmic complexity measures (Solomonoff 1964, Kolmogorov 1965, Chaitin 1966, Martin-Löf 1966), a remarkable number of different definitions of complexity have been suggested over the decades. Classic overviews are due to Lindgren and Nordahl (1988), Grassberger (1989, 2012), or Wackerbauer *et al.* (1994). Though some complexity measures seem to be more popular than others, there are no clear or rigorous criteria to select a "correct" definition and reject the rest.

It appears that for a proper characterization of complexity one of the fundaments of scientific methodology, the search for *universality*, must be complemented by an unavoidable context-dependence, or *contextuality*. An important example for such contexts are the role of the environment, including measuring instruments, in the measurement process of quantum theory (Atmanspacher 1997).[2] Another case in point is the model class an observer has in mind when modeling a complex system (Crutchfield 1994). For a more detailed account of some epistemological background for these topics compare Atmanspacher (1997).

A systematic orientation in the jungle of definitions of complexity is impossible unless a reasonable classification is at hand. Again, several approaches can be found in the literature: two of them are (i) the distinction of structural and dynamical measures (Wackerbauer *et al.* 1994) and (ii) the distinction of deterministic and statistical measures (Crutchfield and Young 1989).[3] Another,

[2] A particularly relevant feature of quantum measurement for the discussion of reproducibility is that successive measurements do not commute. This is due an uncontrollable backreation of the measurement process on the state of the system. This feature is addressed in more detail in the contributions by Collins and by Wang and Busemeyer in this volume.

[3] Note that deterministic measures are not free from statistical tools. The point of this distinction is that individual accounts are delineated from ensemble accounts.

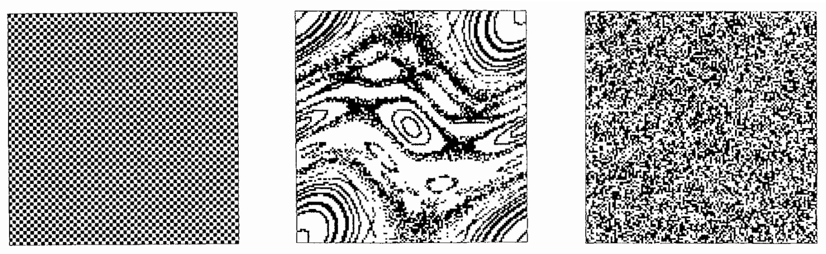

Figure 1: Three patterns used to demonstrate the notion of complexity. Typically, the pattern in the middle is intuitively judged as most complex. The left pattern is a periodic sequence of black and white pixels, whereas the pattern on the right is constructed as a random sequence of black and white pixels. (Reproduced from Grassberger (1986) with permission.)

epistemologically inspired (Scheibe 1973) scheme (iii) assigns ontic and epistemic levels of description to deterministic and statistical measures, respectively (Atmanspacher 1994).

A phenomenological criterion for classification refers to the way in which a complexity measure is related to randomness, as illustrated in Figure 1 (for an early reference in this regard see Weaver 1968).[4] This perspective gives rise to two classes of complexity measures: (iv) those for which complexity increases monotonically with randomness and those with a globally convex behavior as a function of randomness. Classifications according to (ii) and (iii) distinguish measures of complexity precisely in the same manner as (iv) does: deterministic or ontic measures behave monotonically, while statistical or epistemic measures are convex. In other words: deterministic (ontic) measures are essentially measures of randomness, whereas statistical (epistemic) measures capture the idea of complexity in an intuitively appealing fashion.

Examples for monotonic measures are algorithmic complexity (Kolmogorov 1965), various kinds of Rényi information (Balatoni and Rényi 1956), among them Shannon information (Shannon 1949), multifractal scaling indices (Halsey *et al.* 1986), or dynamical entropies (Kolmogorov 1958). Examples for convex measures are effective measure complexity (Grassberger 1986), ϵ-machine complexity (Crutchfield and Young 1989), fluctuation complexity (Bates and Shepard 1993), neural complexity (Tononi *et al.* 1994), variance complexity (Atmanspacher *et al.* 1997).

An intriguing difference (v) between monotonic and convex measures can be recognized if one focuses on the way statistics is implemented in each of them.

[4]It should be emphasized that randomness itself is a concept that is anything else than finally clarified. Here we use the notion of randomness in the broad sense of an entropy.

The crucial point is that convex measures, in contrast to monotonic measures, are formalized *meta*-statistically: They are effectively based on second-order statistics in the sense of "statistics over statistics".[5] Fluctuation complexity is the standard deviation (second-order) of a net mean information flow (first-order), effective measure complexity is the convergence rate (second-order) of a difference of entropies (first-order), ϵ-machine complexity is the Shannon information with respect to machine states (second-order) that are constructed as a compressed description of a data stream (first-order), and variance complexity is based on the global variance (second-order) of local variances of a distribution of data. Monotonic complexity measures provide no such two-level statistical structures.

While monotonic complexity measures are essentially measures of randomness, intuitively appropriate measures of complexity are convex. Corresponding definitions of complexity are highly context-dependent, hence it is nonsensical to ascribe an amount of complexity to a system without specifying the precise context under which it is considered. As a consequence, already the assignment of some degree of complexity is reproducible only if all relevant contexts are explicitly known and taken into account. Since convex complexity measures do not obey a universal definition, strict reproducibility (as, e.g., in classical mechanics) cannot be taken for granted in complex systems.

3 Complexity and Meaning

Grassberger (1986) and Atlan (1987) were the first to emphasize a close relationship between complexity and the concept of meaning (semantic information). For instance, Grassberger (1989, his italics) wrote:

> complexity in a very broad sense is a *difficulty* of a *meaningful task*. More precisely, the complexity of a pattern, a machine, an algorithm etc. is the difficulty of the most important task related to it. ... As a consequence of our insistence on *meaningful* tasks, the concept of complexity becomes *subjective*. We really cannot speak of the complexity of a pattern without reference to the observer. ... A unique definition (of complexity) with a universal range of applications does not exist. Indeed, one of the most obvious properties of a complex object is that there is no *unique* most important task related to it.

[5] "Second-order statistics" does not mean that the second moment of a distribution has to be involved.

Although this remarkable statement by one of the pioneers of complexity research in physics dates almost 30 years back from now, it is still quite unclear how the relation between complexity and meaning looks in detail. Before we come to this, let us look at the notion of meaning in some more detail.

Traditionally, the concept of meaning has been of concern for philosophy, and later psychology and cognitive science. Early in the 19th century, Schleiermacher and Dilthey laid the foundations of what is today known as the "hermeneutic method", and late in the same century Brentano introduced the term "intentionality" as the reference relation that connects a mental representation with what it represents. Another approach, which will be of interest in the following, is due to Peirce and has been given an information theoretical framework by Morris (1955): the "semiotic approach".

Semiotics, the study of signs, is constituted by three different fields: syntactics, semantics, and pragmatics. While the syntactic level is relevant for the interrelations between signs (e.g., grammar), semantics deals with the relation between signs and what they designate (their meaning), and pragmatics focuses on the relation between signs, their meaning, and their users. Applying this to the realm of scientific models, one can distinguish between the syntactic level of the pure formalism of a model, the semantic level of its interpretation, and the pragmatic level of its application. Semantics addresses the meaning of the formalism, and pragmatics addresses its usage.

However, the apparent clarity of this distinction is somewhat artificial. As soon as one starts to consider aspects of constructing, testing, and working with a model concretely, any rigorous demarcation dissolves. Ultimately, syntactics, semantics, and pragmatics are no longer strictly separable (for a more detailed discussion see Atmanspacher 1994, 2007). Nevertheless, their separation remains useful as a tool for conceptual analysis. Within the present context, this allows us to refer to meaning as the central notion of semantics without explicitly incorporating syntactics and pragmatics at the same time.

Weaver's contribution in the seminal work by Shannon and Weaver (1949) indicated early on that the purely syntactical component of information today known as Shannon information requires extension into semantic and pragmatic domains. For instance, imagine a "Babylonian library" of books most of which are meaningless to most readers because their texts merely satisfy some syntactic rules (if at all). Only a small fraction of the syntactic information contained in the library would amount to semantic information for a particular reader.[6]

[6] In this sense, a plausible condition for understanding meaning is that the structural organization of reader and text possesses commonalities. If this is the case, they are not independent, and the mutual information between them is greater than zero. This can be discussed in terms

How is it possible to check if a receiver understood the meaning of a message? Shortly after Shannon and Weaver's work, Bar Hillel and Carnap (1953) proposed a quantification of semantic information based on a receiver's ability to draw valid logical conclusions from a received message. Their approach tries to measure semantic information by its consequences. If meaning is understood, then it triggers action and changes the structure or behavior of the receiver. In this spirit, Weizsäcker (1974) has introduced a way to deal with the usage that is based on the meaning of a message in terms of pragmatic information.[7]

Pragmatic information is based on the notions of *novelty* ("Erstmaligkeit") and *confirmation* ("Bestätigung"). Weizsäcker argued that a message that does nothing but confirm the prior knowledge of a receiver will not change its structure or behavior. On the other hand, a message providing (novel) material completely unrelated to any prior knowledge of the receiver will not change its structure or behavior either, simply because it cannot be understood. In both cases, the pragmatic information of the message vanishes. A maximum of pragmatic information is assigned to a message that transfers an optimal combination of novelty and confirmation to its receiver. Purely syntactic Shannon information represents the limiting case of pragmatic information for complete confirmation. If novelty is added, Shannon information increases monotonically.

Pragmatic information can be made operationally accessible, as has been shown by Gernert (1985) and Kornwachs and Lucadou (1985) who applied pragmatic information to the study of cognitive systems. But also purely physical systems allow (though not require) a description in terms of pragmatic information. This has been shown by Atmanspacher and Scheingraber (1990) for physical systems far from thermal equilibrium. Instabilities in a laser system can be considered as meaningful in the sense of positive pragmatic information if they are accompanied by a change of the degree of complexity of the system.

One may object that this approach does not yield explanatory surplus over a purely physical model of such systems. However, if an explicit account of cognition becomes unavoidable, this objection dissolves. Atmanspacher and Filk (2006) demonstrated that the complexity of networks performing supervised learning behaves non-monotonically as learning proceeds. Their plausible suggestion is to interpret the maximum amount of complexity during the learning process as the point at which the learning task is represented in a way that becomes meaningful for the final solution of the task.

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of monotonic complexity measures. For instance, Chaitin (1979) developed such a "common information" in the framework of algorithmic information theory.

[7]A collection of papers concerning this concept can be found in the journal *Mind and Matter* **4**(2) of 2006, including a more recent account by Weizsäcker and Weizsäcker (2006).

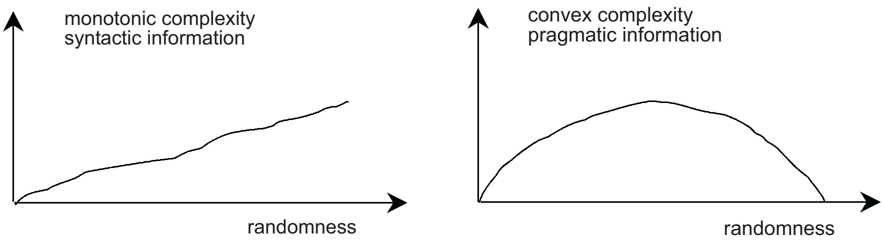

Figure 2: Two classes of complexity measures: Monotonic complexity measures essentially are measures of randomness, typically based on syntactic information. Convex complexity measures vanish for complete randomness and can be related to the concept of pragmatic information.

In a very timely and concrete sense, complexity and meaning are tightly related in a number of approaches in the rapidly developing field of semantic information retrieval as used in semantic search algorithms for databases (for an early reference see Amati and van Rijsbergen 1998, a recent collection of essays is due to de Virgilio *et al.* 2012). Many such approaches try to consider the context of lexical items by similarity relations, e.g. based on the topology of these items (see, e.g., Cilibrasi and Vitànyi 2007). This pragmatic aspect of meaning is also central in situation semantics on which some approaches of understanding meaning in computational linguistics are based (e.g., Rieger 2004).

Based on pragmatic information, an important connection between meaning and complexity can be established (cf. Atmanspacher 1994, 2007). Applying a proper algorithm in order to generate a regular pattern, e.g., a checker-board-like period-2 pattern, the corresponding generation process is obviously recurrent after the first two steps (compare Fig. 1 left). Considering the entire generation process as a process of information transmission, it is clear that any part of the process after its first two time steps just confirms these first steps. In this sense, a regular pattern of vanishing complexity corresponds to a process of information transmission with vanishing meaning as soon as an initial transient phase (the first two time steps) is completed. This argument holds for both monotonic (deterministic) and convex (statistical) definitions of complexity.

For a maximally random pattern the situation is more involved, since monotonic, deterministic complexity and convex, statistical complexity lead to different assessments. *Deterministically*, a random pattern is generated by an incompressible algorithm which contains as many steps as the pattern contains elements. The process of generating the pattern is not recurrent within the length of the algorithm. This means that it never ceases to produce elements that are unpredictable (except the entire algorithm were *a priori* known, such as the sequence of digits of the number π). Hence, the process generating a random pattern can be interpreted as a transmission of information completely lacking confirmation, and consequently with vanishing meaning.

If the *statistical* notion of complexity is focused on, the process of pattern generation is no longer related to the sequence of individually distinct elements. A statistical generation of the pattern is not uniquely specific with respect to its local properties; it is completely characterized by the global statistical distribution. This entails a significant shift in perspective. While monotonic complexity relates to syntactic information as a measure of randomness, the convexity of statistical complexity coincides with the convexity of pragmatic information as a measure of meaning (see Figure 2). It is remarkable how the concepts of complexity and meaning are explicitly complementary in this respect.

Subtle combinations of regular and random behavior in complex systems correspond to subtle combinations of confirmation and novelty in terms of pragmatic information. There is a correspondence between convex complexity measures and pragmatic information as an operational measure of meaning. This correspondence can be understood phenomenologically (by the behavior of those measures as a function of randomness), formally (by their statistical structure), and conceptually (by epistemological arguments). The relation between complexity and meaning is important for learning theory, semantic information retrieval, and computational linguistics.

4 Beyond Stationarity and Ergodicity

Another important set of complications beyond the methodology of conventional science due to complex behavior refers to non-stationary, transient trajectories at instabilities between different modes of stable behavior. In addition, ergodicity cannot be generally presupposed in such cases. Roughly, this means that the ensemble average of a given system variable is no longer identical with its time average along an individual trajectory. For non-ergodic systems, statistical measures from ergodic theory must be carefully scrutinized (Tanaka and Aizawa 1993).[8]

As a consequence, imprudent applications of *limit theorems* in statistical analyses of data from complex systems can lead to pitfalls and misinterpretations. The framework of *large deviations statistics* (LDS), originally applied to problems of statistical physics (Ellis 1985, Oono 1989), has been proposed as a useful tool

[8]Non-stationary and non-ergodic behavior in this sense is expected to play a particularly significant role in living organisms and cognitive systems (cf. Freeman 1994, Nozawa 1994).

for the study of complex systems (Aizawa 1989, Seppäläinen 1995).[9] In cases where it is difficult to see how a system variable behaves, LDS allows us to estimate how long its convergence toward a limit will take.

If a variable is defined in the sense of an expectation value, then the relevant framework is that of a level-1 LDS description. The expectation value is defined in the limit of $N \to \infty$, e.g., in a "thermodynamic" limit, where N can be the number of particles, of degrees of freedom, of subensembles, etc. For instance, the formalism of multifractal measures (Halsey *et al.* 1986, Paladin and Vulpiani 1987) is based on a thermodynamic limit; it is a level-1 theory and uses such first-order statistical measures.

The *law of large numbers* states that in the appropriate limit a distribution converges weakly to the unit point measure at the expectation value. LDS specifies how fast it converges in terms of an (exponential) *convergence rate*, the large deviation entropy (Ellis 1985). A more restrictive limit theorem which (other than a law of large numbers) presupposes the existence of the second moment of a distribution is the *central limit theorem*. It gives an estimate for the probability that the size of properly defined, i.e., normalized, fluctuations around the expectation value is of the order of $\sqrt{N}$.[10]

If the thermodynamic limit as a precondition for a law of large numbers in the sense of a level-1 description cannot be presupposed, one can consider a higher level at which the observed empirical distribution functions themselves (not single variables) are treated as stochastic objects. Measures on such a higher level are meta-statistical measures; they characterize the fluctuations of a distribution as a function of N. Distributions in a purely structural (non-dynamical) sense give then rise to meta-statistical level-2 descriptions.

A good example is the behavior of histograms of scaling indices for finite N as a function of N, which become multifractal measures for $N \to \infty$ (cf. Halsey *et al.* 1986). For distributions covering structural as well as dynamical elements it can be reasonable to proceed to meta-statistical descriptions that are called level-3 descriptions in the terminology of LDS (Ellis 1985). The stochastic objects of these descriptions are trajectories or histories instead of level-2 distributions.

A level-$(n-1)$ theory can in general be obtained from the corresponding level-n theory ("contraction principle"; Ellis 1985). For instance it is possible

[9]Relationships between LDS and ϵ-machine reconstruction in computational mechanics have been indicated by Young and Crutchfield (1994). A most up-to-date collection of LDS applications in physics is due to Vulpiani *et al.* (2014).

[10]Some background on standard limit theorems in statistics can be found in the contribution by Stahel, this volume. The extreme case of rare events, where the limit $N \to \infty$ is not achievable by definition, is treated by Kantz, this volume.

to infer the convergence rate toward an expectation value (assuming that it exists) from the convergence rate of its probability distribution. An analogous contraction principle does not in general apply to the *moments*. If the distribution function depends on time, ergodicity may break down so that ensemble averages cannot be expressed as time averages. If this is not explicitly taken care of, this can lead to severe problems with respect to the law of large numbers.

In a pertinent example, Pikovsky and Kurths (1994) clarified a corresponding misunderstanding in numerical studies of globally coupled maps (see also Griniasty and Hakim 1994). Coupled maps are networks of recursive nonlinear maps, such as the logistic map, which have been used to study the spatiotemporal properties of commplex systems. Coupled map lattices can be seen as discrete-time simulations of partial differential equations. Pioneering work in this direction is due to Cruthfield, Kaneko and others in the early 1980s.

Pikovsky and Kurths (1994) showed that properly defined higher-order fluctuations are consistent with the law of large numbers at level-3, although they violate it at level-2 (Kaneko 1990). The reason is that stationarity and ergodicity break down in complex systems such as coupled map lattices or, more specifically, globally coupled maps (see also Aizawa 1989).

This is particularly interesting in view of the fact that such systems generically give rise to *long-living transients* – a field of research pioneered by Grebogi *et al.* (1985) and recently reviewed by Tel and Lai (2008). So-called *supertransients* are tightly connected to intrinsically unstable phases during which a system transits from previously stable behavior to a new stable solution. Their lifetimes scale exponentially or algebraically as a function of particular system parameters once these parameters exceed a critical value. Supertransients are frequent in spatially extended systems.

Coupled map lattices also offer an interesting playground for how intrinsically unstable behavior can be stabilized – a kind of "non-hierarchical control" due to the crucial dependence of each site in the lattice on its environment, e.g. its neighboring sites. Numerical studies by Atmanspacher *et al.* (2005) showed that this kind of feedback may stabilize the behavior of a system exactly in its *unstable* regime (e.g., at unstable fixed points). This counterintuitive result differs fundamentally from standard ways of controlling chaos by an ongoing external adjustment of parameters (Ott *et al.* 1990).

Basic assumptions in the statistical analysis of systems in traditional science are ergodicity, stationarity, stability, etc. All these assumptions can be inappropriate in complex systems. Particular examples show that, as a consequence, statistical limit theorems need to be applied with great care, for instance with the help of large deviations statistics. Since limit theorems are a basic statistical *backbone of reproducible results, it is evident that the concept of reproducibility must not be uncritically assumed but carefully checked in each individual situation. Extremely long-living transients between stable behavior may be other indicators of intrinsically irreproducible behavior.*

5 Conclusions

After several decades of research on complexity, there is now much evidence for an intimate relationship between the phenomenological classes of monotonic and convex complexity measures and the first-order and second-order statistical structure of these measures, respectively. Problems with the application of limit theorems (laws of large numbers, central limit theorem) can in a compact and general manner be addressed within the framework of large deviations statistics (LDS). First-order statistics (expectation values) may become irrelevant in systems requiring higher-order statistics (limit distributions, histories) as covered by LDS.

On the other hand, it is well known that first-order limit theorems (for the expectation value of an observable) often are not relevant for the statistical analysis of complex systems. For such systems it becomes mandatory to investigate limits of distributions or even limits of histories of distributions. The connection between convex complexity measures and higher-order LDS thus indicates promising novel directions with respect to a proper formal framework for the study of complex systems.

The significance of these ideas is further supported by a number of important epistemological issues (Atmanspacher 1994, 1997). Today it is a truism that the complexity of a system cannot be uniquely characterized unless an observer's model of this system is *explicitly* taken into account.[11] This implies that complexity is not a property of a system "out there" but rather a property of the relation between a system "out there" and the modeling framework by which it is assessed.

As a straightforward consequence, any model of this relationship itself has (at least) to be a meta-model (see, e.g., Casti 1992). A theory of complexity in this sense must, thus, be a second-order (meta-) theory. Its referents are not merely measured facts or data but *also* first-order models of these data *and* the relation between the two. This relation manifests itself most evidently in the process of model-building, or learning.

[11] See Grassberger (1989), but also Ehm and Stahel, both in this volume. What Collins, also in this volume, addresses as "experimenters' regress" and its consequences can be seen under this aspect as well. This applies in particular to his recommendation for "meta-meta-analysis".

All this entails a profound change of perspective as compared to conventional methodological principles in science. One of the most remarkable points in this regard is the altered situation with respect to the issue of operationalization in a metatheoretical framework. It is obviously inadequate to confirm or reject a meta-model simply by a "naive" observation of a "pure" fact. Instead, proper "experiments" have to include the relationship between data and models.

In general, their analysis has to be a meta-analysis, and in general it has to be based on meta-statistics instead of conventional first-order statistics. The concepts of predictability and reproducibility, which needed critical reconsideration already for nonlinear dynamical, particularly chaotic, systems (see the contributions by Zimmerli and Bailey *et al.*, this volume) will have to come under even more scrutiny in a theory of complex systems.

Acknowledgments

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